

## GAME THEORY #3

### A reminder

(\*) Current topic: Static games with complete information (SGCI)

1 round, players move simultaneously

players have all relevant information

(\*) Elements of SGCI:

Players:  $N = \{1, \dots, n\}$

Actions  $A = A^{(1)} \times A^{(2)} \times \dots \times A^{(n)}$

Payoffs:  $\pi: A \rightarrow \mathbb{R}^n$

(\*) Strategies for SGCI

Probability distributions  $\sigma = (\sigma_1^{(i)}, \dots, \sigma_k^{(i)})$

$\sum^{(i)}$  ... set of all strategies of player i

$\Sigma = \Sigma^{(1)} \times \Sigma^{(2)} \times \dots \times \Sigma^{(n)}$  strategy profiles

Extend payoff function such that  $\pi: \Sigma \rightarrow \mathbb{R}^n$

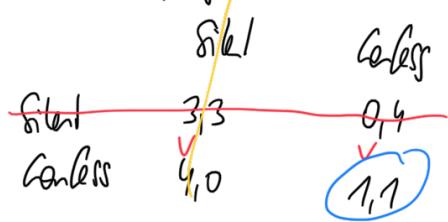
Pure strategy  $s_j^{(i)}$ : Player i plays  $a_j^{(i)}$  with probability 1

(\*) Our task now: Given we have a game.

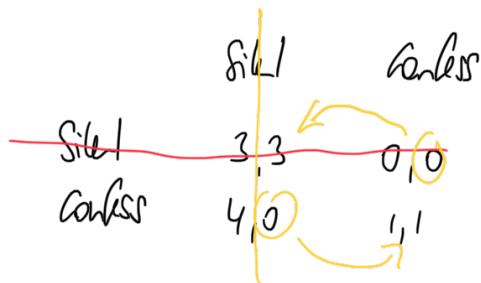
What would be a reasonable solution?

Absolutely:  $\Psi: \Pi = (N, A, \pi) \rightarrow \Sigma$

(\*) Special case of prisoner's dilemma



Example 2.13 (Prisoner's dilemma with remorse)



Definition 2.14 (Iterated elimination of dominated strategies, IEDS)

Define recursively

$$S_0^{(i)} = S^{(i)}, \quad \Sigma_0^{(i)} = \Sigma^{(i)}$$

Define recursively all strategies that are not dominated after  $k$  steps

$$S_k^{(i)} = \left\{ s^{(i)} \in S_{k-1}^{(i)} : \nexists \sigma^{(i)}: \pi^{(i)}(\sigma^{(i)}, s^{(i)}) > \pi^{(i)}(s^{(i)}, s'^{(i)}) \right\}$$

$$\Sigma_k^{(i)} = \left\{ \sigma^{(i)} \in \Sigma^{(i)} : \sigma_j^{(i)} > 0 \Rightarrow s_j \in S_k^{(i)} \quad \forall s^{(i)} \in S_{k-1}^{(i)} \right\}$$

We call the game dominance solvable if

for all players  $i$  the object  $\bigcap_{k=0}^{\infty} S_k^{(i)}$  contains only one element.

Example 2.15 (Prisoner's dilemma with remorse)

Silent      Confess

Silent	<u>3,3</u>	<u>0,0</u>
Cheating	4,0	1,1

<u><math>k=0</math></u>	$S_0^{(1)} = \{ \text{Silent, Cheating} \}$	$S_0^{(2)} = \{ \text{Silent, Cheating} \}$
<u><math>k=1</math></u>	$S_1^{(1)} = \{ \text{Cheating} \}$	$S_1^{(2)} = \{ \text{Silent, Cheating} \}$
<u><math>k=2</math></u>	$S_2^{(1)} = \{ \text{Cheating} \}$	$S_2^{(2)} = \{ \text{Cheating} \}$
$k > 2$	— (1) —	— (1) —
	$\bigcap_{k=0}^{\infty} S_k^{(1)} = \{ \text{Cheating} \}$	$\bigcap_{k=0}^{\infty} S_k^{(2)} = \{ \text{Cheating} \}$

### Example 2.16 (Traveler's dilemma)

An airline needs to reimburse two travelers for having lost their (identical) suitcases.

Each traveler needs to say a value  $\{180, \dots, 300\}$

If travelers say different amounts, the one with the lower amount gets some reward  $R = 5$ .

Game:  $N = \{ \text{Traveler 1, Traveler 2} \}$   $[180, 300] \in X$

$$A^{(1)} = \{ 180, \dots, 300 \}$$

$$\pi^{(1)}(\alpha_1, \alpha_2) = \begin{cases} \alpha_1 & \text{if } \alpha_1 = \alpha_2 \\ \alpha_1 + R & \text{if } \alpha_1 < \alpha_2 \\ \alpha_2 & \text{if } \alpha_1 > \alpha_2 \end{cases}$$

$$\underline{k=0} \quad S_0^{(1)} = \{ 180, 181, \dots, 300 \} \quad S_0^{(2)} = \{ 180, 181, \dots, 300 \}$$

$\omega_0 = 1184, \dots, \omega_{ij}$

$k=1$  Claim:  $\omega = 300$  is (weakly) dominated by  $299$

Proof: Case 1: Suppose co-player chooses  $\omega_2 \leq 299$

$$\Pi^{(1)}(300, \omega_2) = \omega_2 = \Pi^{(1)}(299, \omega_2)$$

Case 2: Suppose co-player chooses  $\omega_2 = 300$

$$\Pi^{(1)}(300, 300) = 300$$

$$\Pi^{(1)}(299, 300) = \overbrace{304}^{\wedge}$$

$$S_1^{(1)} = \{180, \dots, 299\} \quad S_1^{(2)} = \{180, \dots, 299\}$$

$$\underline{k=2} \quad S_2^{(1)} = \{180, \dots, 299\} \quad S_2^{(2)} = \{180, \dots, 298\}$$

Arbitrary  $k \leq 120 \quad S_k^{(1)} = \{180, \dots, 300-k\}$

$$k > 120 \quad S_k^{(1)} = \{180\}$$

IEDS is complex as 2 more examples in **exercises**

Remark 2.17 (On the status of IEDS within classical game theory)

- (1) If a game is dominance solvable, the solution is usually considered as quite convincing.

Only requires Rationality + Common knowledge of rationality ✓

(2) Solutions are unique ✓

(3) Solutions may not exist ✗

## Remark 2.18 (IEDS in other game theories)

### (\*) Epistemic game theory

$R$  ... players are related

$\chi R$  ... players know they are related

$\chi^2 R$  ... players know they know they are related

To solve the knowledge dilemmas you need  $\chi^{ip} R$

### (\*) Evolutionary game theory

Replicator dynamics: infinite population, symmetric game

with  $k$  pure strategies,  $x_j$  ... fraction of players using strategy  $s_j$

$$\dot{x}_j = x_j \left[ \pi_j - \bar{\pi} \right]$$

$\nwarrow$  Average

Theorem: Suppose the game is dominance solvable

such that  $s_j$  is the solution

then  $x_j(t) \xrightarrow{t \rightarrow \infty} 1$  as all solutions of replicator dynamics with  $x_j(0) > 0$

Hofbauer & Sandholm (2011) cool!

↳ Readings L13/

### (\*) Behavioral game theory

Goeree & Holt (AER 2001)

Traveler's dilemma  $\{180, 181, \dots, 300\}$  [cents]

$$R = 180 \quad \approx 80\% \text{ w/ } G = 180$$

$$R = 5 \quad \approx 80\% \text{ w/ } G = 300.$$

## § 2.2 Nash equilibria

$$\Psi: \Pi \rightarrow \Sigma$$

### Example 2.19 (Stop-trust)



This is not dominant solvable.

Somewhat irresponsible to call  $(\text{Slap}, \text{Hate})$  a solution.

There is a sense in which  $(\text{Stop}, \text{Stop})$  seems more reasonable.

### Definition 2.20 (Nash equilibrium)

- (1) Consider a game  $\Pi = (\mathcal{N}, \mathcal{A}, \pi)$ . Then a strategy profile  $\hat{\sigma}$  is a Nash equilibrium if for all players  $i$
- $$= (\hat{\sigma}^{(1)}, \hat{\sigma}^{(2)}, \dots, \hat{\sigma}^{(n)})$$
- $$\pi^{(i)}(\hat{\sigma}^{(1)}, \hat{\sigma}^{(2)}, \dots, \hat{\sigma}^{(i)}, \dots, \hat{\sigma}^{(n)}) \geq \pi^{(i)}(\sigma_i, \hat{\sigma}^{(2)}, \dots, \hat{\sigma}^{(n)})$$

$$\sum_{i=1}^n \hat{G}_i = G$$

(2) The NE is called strict if the inequality is strict  
for all  $\hat{G}^{(i)} \neq \hat{G}^{(j)}$